Anomalous Viscosity, Resistivity, and Thermal Diffusivity of the Solar Wind Plasma

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February 5, 2008

Abstract

In this paper we have estimated typical anomalous viscosity, resistivity, and thermal difffusivity of the solar wind plasma. Since the solar wind is collsionless plasma, we have assumed that the dissipation in the solar wind occurs at proton gyro radius through wave-particle interactions. Using this dissipation length-scale and the dissipation rates calculated using MHD turbulence phenomenology [*Verma et al.*, 1995a], we estimate the viscosity and proton thermal diffusivity. The resistivity and electron's thermal diffusivity have also been estimated. We find that all our transport quantities are several orders of magnitude higher than those calculated earlier using classical transport theories of *Braginskii*. In this paper we have also estimated the eddy turbulent viscosity.

1 Introduction

The solar wind is a collisionless plasma; the distance travelled by protons between two consecutive Coulomb collisions is approximately 3 AU [*Barnes,* 1979]. Therefore, the dissipation in the solar wind involves wave-particle interactions rather than particle-particle collisions. For the observational evidence of the wave-particle interactions in the solar wind refer to the review articles by *Gurnett* [1991], *Marsch* [1991] and references therein. Due to these reasons for the calculations of transport coefficients in the solar wind, the scales of wave-particle interactions appear more appropriate than those of particle-particle interactions [*Braginskii*, 1965].

Note that the viscosity in a turbulent fluid is scale dependent. The viscosity discussed in most part of this paper is the one at the dissipation length-scale, not at the large or intermediate length-scales. The viscosity at large scale is called turbulent eddy viscosity; it is briefly discussed at the end of section 2 of this paper. In fluid turbulence, the viscosity at dissipation scales is determined from the dissipation rate and the dissipation length scale [*Lesieur,* 1989]. In this paper we estimate viscosity, resistivity, and thermal conductivity of the solar wind using similar technique. The dissipation length-scales is obtained from the wave-particle interactions, and the dissipation rates are obtained from the Kolmogorov-like MHD turbulence phenomenology [*Marsch,* 1990; *Matthaeus and Zhou,* 1989; *Zhou and Matthaeus,* 1990]. For the solar wind *Tu* [1988] and *Verma et al.* [1995a] have calculated the dissipation rates from the observed energy spectra using the Kolmogorov-like MHD turbulence phenomenology. Since in our approach the wave-particle interactions dominate the particle-particle collisions, the transport coefficients presented here are the anomalous transport coefficients commonly referred to in fusion plasma literature. Note that the transport quantities in the solar wind vary with distance. In this paper we estimate these quantities at 1 AU.

Earlier *Montgomery* [1983] has calculated the transport coefficients in the solar wind using the *Braginskii*'s [1965] formalism, which is based on particle-particle collision. He found both kinematic viscosity and resistivity to be of the order of 10^{-6} km^2 s^{-1}. Using the velocity of the large eddies as 20 km s^{-1} and length-scale as 10^6 km, he obtained the Reynolds number to be of the order of 10^{13}. Note that *Montgomery* [1983] used η_1 of *Braginskii* rather than η_0. This is consistent with the *Hollweg*'s result [1985] where he showed

that the η_0 terms are fully accounted for by the diagonal pressure tensor.

In the following section we will estimate the length-scale at which wave-particle interactions take place. This will be the dissipation length-scale for our calculation of the transport coefficients. Using the dissipation rates calculated earlier, we then estimate the kinematic viscosity, resistivity, and thermal diffusivity at dissipation length-scales. Towards the end of section 2, we have also estimated the eddy viscosity and thermal diffusivity of the solar wind. Section 3 contains conclusions.

2 Calculation

Verma et al. [1995a] have calculated the dissipation rates in the solar wind streams using the Kolmogorov-like MHD turbulence phenomenology [*Marsch,* 1990; *Matthaeus and Zhou,* 1989; *Zhou and Matthaeus,* 1990]. The choice of this phenomenology over the Kraichnan's phenomenology [*Kraichnan,* 1965] or the Dobrowolny et al.'s generalization of Kraichnan's phenomenology [*Dobrowolny et al.,* 1980] is motivated by the fact that the observed solar wind energy spectra tend to be closer to Kolmogorov's $k^{-5/3}$ power law than Kraichnan's $k^{-3/2}$ power law. Also, temperature evolution study of *Verma et al.* [1995a] show that the predictions of the temperature evolution using the Kolmogorov-like model are in closer agreement with the observations than those using Kraichnan's or Dobrowolny et al.'s models. Also refer to *Tu* [1988] for theoretical studies of turbulent heating in the solar wind.

The Kolmogorov-like phenomenology provides the energy spectra of fluctuations $\mathbf{z}^{\pm} = \mathbf{u} \pm \mathbf{b}/\sqrt{4\pi\rho}$, where $\mathbf{u}$ is the velocity field fluctuation, $\mathbf{b}$ is the magnetic field fluctuation, and ρ is the density of the plasma. The quantities $\mathbf{z}^{\pm}$ represent the amplitudes of Alfvén waves having positive and negative velocity-magnetic field correlations respectively. The energy spectra according to this phenomenology are

$$E^{\pm}(k) = K^{\pm} \left(\epsilon^{\pm}\right)^{4/3} \left(\epsilon^{\mp}\right)^{-2/3} k^{-5/3}. \tag{1}$$

where $\epsilon^{\pm}$ are the dissipation rates of $\mathbf{z}^{\pm}$ fluctuations, and $K^{\pm}$ are Kolmogorov's constants for MHD turbulence. According to *Verma et al.* [1995a] the dissipation rates of the solar wind streams are of the order of 10^{-3} km^2 s^{-3}.

As mentioned in the introduction, we estimate the dissipation length-scale from the theories of wave-particle interactions. It has been shown that the wave-particle resonance between MHD waves and ions occurs either in the form of the Doppler-shifted cyclotron resonance,

$$\omega - k_{\parallel} v_{\parallel} = n\Omega_i; (n = \pm 1, \pm 2, \cdots) \tag{2}$$

or in the form of the Landau resonance

$$\omega - k_{\parallel} v_{\parallel} = 0, \tag{3}$$

where Ω_i is the cyclotron frequency of the ions, ω is the wave frequency, $k_{\parallel}$ and $v_{\parallel}$ are the parallel components along the mean magnetic field of the wave number and the ion velocity vector respectively [*Stix,* 1962; *Barnes,* 1979]. For the solar wind, at 1 AU the cyclotron frequency Ω_i of the ions is of the order of 1.0 s^{-1}, and the thermal speed v is of the order of 50 km s^{-1} [*Barnes,* 1979]. Typical Alfvén speed ω/k at 1 AU is also 50 km s^{-1}. Also note that the solar wind fluctuations are dominated by Alfvén waves; the compressive waves are damped at the early stages of its transit.

The process by which Alfvén waves might be damped have been the subject of considerable research. Since $\omega/k \sim 50$ km/s $\sim v$, it appears that Alfvén waves can be Landau damped. However, *Barnes and Suffolk* [1971] and *Barnes* [1979] argue against this. They show that the transverse Alfvén waves are exact solutions of the Vlasov-Maxwell equations for arbitrary amplitude, hence it can not be damped. But that is not correct either. It has been shown that all hydromagnetic waves, except the Alfvén mode with precise circular polarization, steepen and evolve into other modes or collisionless shocks [*Tidman and Krall*, 1971]. *Sagdeev and Galeev* [1969] showed that a linearly polarized Alfvén wave is unstable and it decays to a back scattered Alfvén wave and magnetosonic waves. The magnetosonic waves thus generated get damped by Landau damping (see *Barnes* [1979] and references therein for discussion on Landau damping of magnetosonic waves). *Hollweg* [1971] has obtained similar results. Hence the Alfvén waves in the solar wind can get damped by decaying to a magnetosonic waves which in turn gets damped by Landau damping.

Now the question arises, which waves in the solar wind are affected by the above process. The energy from the small and intermediate k (large wavelength) waves cascades to larger k waves due to nonlinear interaction

arising from the $\mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm}$ term of MHD equation [*Kraichnan,* 1965], and these waves do not get damped. At the dissipation scale the energy cascade stops. We conjecture that the decay of Alfvén waves to magnetosonic waves, and the damping of the generated magnetosonic waves occur near the ion gyro radius $r = 100$ km. Therefore, $k_d = 10^{-2}$ km^{-1}.

Regarding the cyclotron resonance, the small k Alfvén waves of the solar wind cannot be damped by this mechanism because $w \ll \Omega_i$ and $kv \ll \Omega_i$, when k is small [see Eq. (2)]. However, when k becomes large, it is possible for the waves to get damped by cyclotron damping. The approximate k where cyclotron resonance could happen is

$$k_d \sim \frac{\Omega_i}{V_A - V_{\parallel}} \sim \frac{1}{50km} \sim 10^{-2} km^{-1}. \tag{4}$$

Hence the dissipation length-scale for the particle-wave interaction is approximately 100 km, and the dissipation wavenumber is $k_d = 10^{-2}$ km^{-1}. The solar wind observations show that at 1 AU the transition from inertial range to dissipation range occurs at around a length scale of 400 kms [*Roberts,* 1995], a result consistent with our above arguments. In this paper we assume that the dissipation length-scales for fluid energy, magnetic energy, and the energy of the Alfvén waves are all same (see Appendix).

In the appendix we derive an expression for the viscosity ν in terms of dissipation rate ϵ and dissipation length-scale (k_d^{-1}) that is

$$\nu \sim \left(\frac{\epsilon}{k_d^4} \right)^{1/3}. \tag{5}$$

Here we assumed that the fluid and magnetic energies are approximately equal, and also that $E^+(k) \sim E^-(k)$. Under this condition $\nu \sim \lambda$. We use this formula for our estimation of viscosity in the solar wind. Substitution of $\epsilon = 10^{-3}$ km^{2}s^{-3} and $k_d = 10^{-2}$ km^{-1} in the above equation yields $\nu \sim 50$ km^2 s^{-1}. This result is very different from the one obtained by *Montgomery* [1983]. Note that the above viscosity is the ion viscosity. It is interesting to note that our estimate of ion viscosity is close to Bohm's diffusion coefficient [*Chen,* 1974], which is

$$D_B = \frac{k_B T c}{16 e B} \sim 100\ km^2 s^{-1} \tag{6}$$

where k_B is the Boltzmann constant, T is the proton temperature, c is the speed of light, e is the electronic charge, and B is the mean magnetic field. The Reynolds number with $\nu = 50$ km^{2}s^{-1}, the mean speed $U = 20$ km s^{-1}, and the length-scale of 10^7 km is

$$Re = \frac{UL}{\nu} = 4 \times 10^6. \tag{7}$$

The dissipation time-scale is

$$\tau_d \sim \frac{1}{k_d v_d} \sim \frac{1}{(k_d^2 \varepsilon)^{1/3}} \sim 200 \text{ s} \tag{8}$$

where v_d is the velocity at the dissipation scale. For the above expression, we assumed that the Kolmogorov-like MHD turbulence phenomenology (Eq. (1)) is valid till $k = k_d$, therefore, $v_d \sim (E(k_d)k_d)^{1/2} \sim (k_d/\epsilon)^{1/3}$ [*Lesieur*, 1989].

Now we estimate electron viscosity and resistivity. The classical electron viscosity ν_e will be [*Braginskii*, 1965]

$$\nu_e = \left(\frac{m_e}{m_i}\right)^{1/2} \nu_i^1 \sim 2 \times 10^{-8} km^2 s^{-1}. \tag{9}$$

Scudder and Olbert [1979a, b] have shown that classical collision transport applies partly to the solar wind 'core' electrons, but are inappropriate for the 'halo' electrons which are affected by whistler waves. However, *Schwartz et al.* [1981] showed that waves with frequencies near the ion gyrofrequencies and wave vectors comparable with inverse ion Larmor radii can provide strong electron-wave coupling. Since at this moment the results are not conclusive, we are following *Schwartz et al.*'s [1981] paradigm.

In the following discussion we will attempt to define and estimate resistivity and electron viscosity in a turbulent plasma. The arguments presented here is over simplified and speculative, and they are in similar lines as that of *Priest* [1982]. However, we believe that these arguments shed some light into this complicated problem and will be useful for future development in this area. The arguments follow: Eddy or kinematic viscosity can be interpreted as diffusion coefficient for coherent fluid parcels. The dissipation length-scale discussed in this paper is the length-scale where the smallest coherent fluid parcel disperse, i.e., fluid energy after this scale is zero (refer to Appendix).

Similarly, the coherent magnetic structures are destroyed by resistivity at the dissipation scales. Since the resistivity is dominated by electron's transport properties, here we estimate electron resistivity. Also since the solar wind plasma is turbulent, we use the length-scales of interactions of the electrons with the waves as the relevant length-scale for this purpose. We assume in this paper that the dissipation length-scales of both fluid and magnetic energy are k_d^{-1}. Therefore, $l_d^e = k_d^{-1} \sim 100$ km. Since the electrons are lighter particles, they move much faster than protons; we assume that the relevant speed of the electrons at dissipation length-scale is its thermal speed. Taking electron temperature as 10^5 K, $v_d^e = 1000$ km/s. From these two scales, we can obtain the time-scale that is $\tau_d^e \sim l_d^e/v_d^e \sim 0.1$ sec. Our above arguments are in the same spirit as that of *Priest* [1982] who has obtained a formula for anomalous conductivity using anomalous collision-time.

Using the above electron velocity and length scales we obtain the electron's kinematic viscosity that is $\nu \sim v_d^e l_d^e \sim 10^5$ km^{2}s^{-1}. We can also estimate the resistivity using the above estimates of length and time scales. The resistivity λ is defined as [*Braginskii,* 1965]

$$\lambda = \frac{m_e c^2}{4\pi n e^2 \tau_d^e} \tag{10}$$

where m_e is the mass of the electron. Substitution of $n = 5$ ions/cc and $\tau_d^e = 0.1$ sec in the above expression yields $\lambda \sim 100$ km^{2}s^{-1}. The resistivity calculated here is close to the resistivity calculated in the earlier part of the paper using the dissipation rates, hence our calculations appear consistent. The magnetic Reynolds number will be

$$Re_M = \frac{UL}{\lambda} \sim 2 \times 10^6. \tag{11}$$

The solar wind magnetic Prandtl number, defined as ν/λ, appears to be of the order of unity. It is interesting to note that both renormalized viscosity $\nu(k)$ and resistivity $\lambda(k)$ are expected to scale as $\epsilon^{1/3}k^{2/3}$, where ϵ is the relevant dissipation rate, and k is the wavenumber [*Verma and Bhattacharjee,* 1995b and references therein]. Therefore, renormalized magnetic Prandtl number $\nu(k)/\lambda(k) \sim 1$. In this paper we are calculating $\lambda(k_d)$ and $\nu(k_d)$. It is reasonable to expect that Kolmogorov's 5/3 power law continues till $k = k_d$, therefore, it is not surprising that our magnetic Prandtl number $\lambda(k_d)/\nu(k_d)$

$\sim$ 1. However, since the above numbers are only order of magnitude estimates, we can not make definite prediction about the magnetic Prandtl number.

The anomalous thermal diffusivity $\kappa_{i,e}$ of ions and electrons of the heat diffusion equation [*Priest,* 1982; *Landau,* 1987]

$$\frac{\partial T_{i,e}}{\partial t} = \kappa_{i,e} \nabla^2 T_{i,e} \tag{12}$$

can also be estimated from the above dissipation length and time scales. It has been argued earlier that heat conduction is carried by superthermal electrons [*Marsch,* 1991 and references therein]. Also, observational studies by *Philip et al.* [1987] show that the heat flux is not proportional to the electron temperature gradient but are regulated by whistler-mode instability driven by the skewness of the distribution function [*Gary et al.*, 1994]. However, here we use turbulence scaling arguments because of the reasons stated above. Since some of the issues are not settled yet, e.g., the heat flux calculated by classical transport quantities are not in good agreement with the observed heat flux, we estimate turbulent thermal diffusivity to provide another point of view.

Here again we use wave-particle interaction time-scale rather the particle-particle collision time-scales. By dimensional arguments the coefficient $\kappa_{i,e}$ can be approximated by

$$\kappa_{i,e} \sim \frac{1}{k_d^2 \tau_{i,e}} \tag{13}$$

The substitution k_d and $\tau_{i,e}$ of the solar wind in the above equation yields $\kappa_i \sim 50$ km^2 s^{-1} and $\kappa_e \sim 10^5$ km^2 s^{-1}. These numbers are same as the viscosity calculated above. The ratio ν/κ is called Prandtl number, and it is of the order unity. Similar to viscosity, the thermal diffusivity calculated here are orders of magnitude higher than the one calculated from the Braginskii's formalism in which $\kappa_i \sim 10^{-6}$ km^2 s^{-1} and $\kappa_e = \kappa_i (m_e/m_i)^{1/2} \sim 2\times10^{-8}$ km^2 s^{-1}(same as the viscosity of *Montgomery* [1983]).

As mentioned in the introduction, viscosity is scale-dependent. The large-scale viscosity, called eddy viscosity, is $\nu_L \sim v_L L$, where L is the large-scale length and v_L is the large-scale fluctuating speed. Therefore, for the solar wind $\nu_L \sim 20km/s \times 10^8 km \sim 10^9 km^2/s$. This number is seven orders of magnitude higher than the viscosity at dissipation length-scale. The thermal diffusivity at large-scales is approximately equal to the eddy viscosity. These

quantities could be useful for the study of solar wind evolution of energy and heat flux etc.

3 Conclusions and Discussion

In this paper we have calculated the viscosity, resistivity, and thermal diffusivity of the solar wind using nonclassical approach. The solar wind is collisionless, therefore, the wave-particle interactions become important while considering dissipation mechanisms in the wind. In this paper we have fixed the dissipation length-scale at proton gyro radius ($\sim$ 100 kms), scale at which wave-particle interactions are expected to occur. This result is consistent with the solar wind observations in which the transitions from inertial range to dissipation range at 1 AU occur at around 400 kms. In our calculation we also need turbulent dissipation rates occurring in the solar wind. In this paper we take the turbulent dissipation rates calculated by *Verma et al.* [1995a].

We find that a typical ion viscosity is 50 km^2 s^{-1} and electron viscosity is 2×10^5 km^2 s^{-1}. The corresponding Reynolds number (with ion viscosity) is around 10^6. The resistivity is around 200 km^2 s^{-1}, and the magnetic Reynolds number is also around 10^6. The magnetic Prandtl number is order unity. The ion and electron thermal diffusivities are the same as the ions and electron viscosities respectively. The large-scale (eddy) viscosity of the wind is approximately 10^9 km^2/s.

All the transport quantities calculated by us are several orders of magnitude higher than those calculated earlier using classical transport theory of *Braginskii.* These results should have important consequences on modelling of the solar wind. Our results show that the thermal diffusivity in the solar wind is much higher than what have been assumed earlier and should be important for the studies regarding the temperature evolution of the solar wind.

The author thanks J. K. Bhattacharjee, D. A. Roberts, M. L. Goldstein, and J. F. Drake for discussion and comments.

In this appendix we derive an expression for viscosity in terms of energy dissipation rates and dissipation length-scales. We use energy equation to derive this expression. Incompressible MHD equation in absence of a mean

magnetic field is [*Kraichnan*, 1965]

$$\frac{\partial}{\partial t}\mathbf{z}^{\pm} = -\mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} - \nabla p + \nu_{+} \nabla^2 \mathbf{z}^{\pm} + \nu_{-} \nabla^2 \mathbf{z}^{\mp} \tag{14}$$

$$\mathbf{z}^{\pm} = \mathbf{u} \pm \mathbf{b} \tag{15}$$

$$\nu_{\pm} = \frac{1}{2}(\nu \pm \lambda) \tag{16}$$

where $\mathbf{u}$ is the fluctuating velocity field, $\mathbf{b}$ is the fluctuating magnetic field in velocity units, p is the total pressure, ν is the kinematic viscosity, and λ is the resistivity. From this equation, under the assumption of isotropy of fluctuations, one can derive [*Orszag,* 1977]

$$\frac{\partial}{\partial t}E^{\pm}(k) = -2\nu_{+}k^2 E^{\pm}(k) - 2\nu_{-}k^2(E^u(k) - E^b(k)) + T^{\pm}(k) \tag{17}$$

where $E^{\pm}(k)$ are the energy spectra of $\mathbf{z}^{\pm}$, $E^u(k)$ and $E^b(k)$ are the velocity and magnetic field energy spectra respectively, and $T^{\pm}(k)$ comes from non-linear term and involves triple correlations of $\mathbf{z}^{\pm}$. By integrating the above equation over the whole spectrum, we obtain

$$\epsilon^{\pm} = -2\nu_{+}\int_0^{\infty} k^2 E^{\pm}(k)dk - 2\nu_{-}\int_0^{\infty} k^2(E^u(k) - E^b(k))dk. \tag{18}$$

The term $T^{\pm}(k)$ upon integration over the whole spectrum yields zero [*Orszag,* 1977].

We make several assumptions to get an order of magnitude estimates of ν. We assume that the third term of the above equation vanishes. This condition will be satisfied either if $\nu_{-} = 0$ or $E^u(k) = E^b(k)$. Since the spectra $E^{\pm}(k)$ is usually strongly damped in the dissipation range, most contribution to the first integral of the above equation comes from k in the range of 0 to k_d. We also make a drastic assumption that $E^{+}(k) = E^{-}(k)$, and $\epsilon^{+} = \epsilon^{-} = \epsilon$. These assumptions are justified only because we are making order of magnitude estimation of ν. To obtain somewhat precise values of $\nu_{\pm}$, we will have to analyse Eq. (18) carefully. Now the substitution Eq. (1) for $E^{\pm}(k)$ in Eq. (18) yields

$$\nu \sim \lambda \sim \nu_{+} \sim \left(\frac{\epsilon}{k_d^4}\right)^{1/3} \tag{19}$$

Hence, given the dissipation rate ϵ and the dissipation length-scale k_d^{-1}, we can estimate ν.

Here we state another assumption which is used in section 2 of this paper. We assume that the dissipation length-scale for all the energies, i.e., $E^{\pm}(k), E^u(k)$, and $E^b(k)$, are the same and are equal to k_d^{-1}.

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